An Integrated Distribution System for Deteriorating Items via an Artificial Intelligence Method

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***Abstract*—This paper proposes a *n*-manufacturer-*one*-distributor-*n*-retailer single-period inventory model for deteriorating items that integrates three levels of distribution system. In order to achieve long-term benefits and global optimum of the system, the different facilities develop their partnership through information sharing and strategic alliances. The mathematical model describes how the integrated approach to decision making can achieve global optimum as compared to independent decision by the manufacturer, the distributor, and the retailer. Due to the complexity of the non-linear problems, it is not possible to find the global optimum analytically. Annealing is the physical process of heating up a solid until it melts followed by cooling it down until it crystallizes into a state with perfect lattice. Following this physical phenomenon, an Artificial Intelligence method—Simulated Annealing (SA), has been developed to find the global optimum for a complex cost surface through stochastic search process. A computer program in C-language has been developed for this purpose and is implemented to derive the optimum decision for the decision maker. Numerical examples and a sensitivity analysis are given to validate the results of the system. The proposed model has potential application in product distribution inventory systems.**

Keywords*—N*-manufacturer-*one*-distributor-*n*-retailer; Single-period inventory*; Deteriorating items; Distribution system; Simulated annealing.*

1. Introduction

Marketing channels are behind every product and service that consumers and business buyers purchase everywhere. Yet in many cases, these end-users are unaware of the richness and complexity necessary to deliver what might seem like everyday items to them. Usually, combinations of institutions specializing in manufacturing, distributor, retailing, and many other areas join forces in marketing channels.

In every channel, three fundamental stages of marketing channel, procurement, production and distribution, have existed independently as disconnected entities, buffered by large inventories. The inventory across the entire chain should be closely monitored because Inventory held to satisfy customer demand increases costs, thus decreasing profits.

If we regard the term marketing channel as a buy-and-sell view of the business, both buyers and vendors may hold excessive safety stock of the same products to satisfy their respective customers. Thus, enterprises are forced to develop channel cooperation that can respond quickly to customer needs with maximum service level. An inventory policy of integrated distribution system will lead to a decrease in costs for each player because this integrated policy.

This study discusses how to use integrated distribution system to reach the marketplace with lower cost or more profit approach, and how a framework for analysis can improve the channel decisions made by an executive acting as a channel manager or designer.

2. Literature Review

There has been a growing interest in supply chain management (SCM) in recent years. The supply chain which is also referred to as the logistic network, consists of manufacturers, distribution centers, and retailer outlets, as well as raw materials, work-in-process inventory, and finished goods that flow between the facilities. Quite a lot of researchers have shown interest in this field of study, and many companies also invest large capital in improving their SCM system. Historically, the three key members of the supply chain, manufacturer, distributor and retailer, have been managed independently, buffered by large inventories.

Increasing competitive pressures and decreasing marginal profitability are forcing firms to develop supply chains that can quickly respond to customer needs, and, furthermore, reduce the cost of carrying inventory. Through their coordination, the number of deliveries is derived in cooperation with each other to achieve a minimum overall integrated cost.

[4] were the first authors to consider the echelon stock in inventory research. [2] made an analysis on the integration between the buyer and the supplier by developing a mathematical model with an arborescent structure. [13] classified echelon stock as centralized echelon and decentralized echelon. Other authors such as [5] have developed different models based on the above two classes.

Besides the consideration of cost issue, the risk reduction of material shortage is also important factor in the supply chain management. Through the integration of suppliers, we may avoid material supply shortage in the supplier chain. In past literatures, most of the inventory models assume that an inventory is replenished from a

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single supplier. However, in reality, there often are situations in which more than one supplier is necessary to satisfy a desirable service goal, specifically in the electronic industrial. [18] developed a mathematical model of *N*-supplier inventory systems and derived an optimal order splitting policy. [17] concluded that dual sourcing is needed when material requirement is large or acquisition lead-time is uncertain. [12] presented optimal ordering policies with two-supplier when lead times and demands are all stochastic. In this study, deterioration is assumed to depend on the condition of the on-hand inventory within the whole supplier chain. In order to reduce loss due to deterioration of the products, the members of the supply chain frequently implement a joint decision on the optimal number of deliveries. [6] were the first authors to consider on-going deterioration of inventory. Since then, several researchers have studied deteriorating inventory.

The inventory control of deteriorating items is so complex that it is impossible to derive the optimum solution via analytical approach and thus researchers are forced to apply approximate nonlinear optimization techniques. To overcome these difficulties, Simulated Annealing (SA) algorithms have been used as optimization techniques for decision-making problems. Annealing is the physical process of heating up a solid until it melts followed by cooling it down slowly until it crystallizes into a state with a perfect lattice. Following this physical phenomenon, SA has been developed to find the global optimum for a complicated complex cost surface. In the early 1980s, [8, 9] and independently [3] introduced the concepts of annealing in combinatorial optimization problems. [1] discussed the conditions under which asymptotic convergence of the SA process is guaranteed. Recently SA has been applied in different areas like Traveling Salesman Problem [10], Graph Partitioning Problems [7], Matching Problems [14], Graph Coloring Problems [16], Scheduling Problems [11]. Recently, [19] had proposed an optimal replenishment policy for a deteriorating green product with considering product life cycle cost. [20-23] discussed the deteriorating inventory regard to collaborative, incentive and pricing strategy.

In this paper, a mathematical model of deterioration item is developed to take into account both integrations: a vertical integration of the suppler, the distributor and the retailer and a horizontal integration of the manufacturers and retailers. The integrated cost minimization model is formulated using SA algorithm. The optimum inventory and scheduling period are evaluated. Numerical values and sensitivity analysis is presented to illustrate the theory.

3. Notation and Assumptions

The following notation is used:

$I_{pi}(t)$ Inventory level of manufacturer i at time t, where i= 1, 2,..., I.

$I_d(t)$ Inventory level of distributor at time t.

$I_{rk}(t)$ Inventory level of retailer k at time t, where k= 1, 2,..., K.

C_{pi} The transportation cost for manufacturer i, where i= 1, 2,..., I.

C_{di} The ordering cost for the distributor, where i= 1, 2,..., I.

C_{fk} The transportation cost for distributor, where k= 1, 2,..., K.

C_{rk} The ordering cost for retailer k, where k= 1, 2,..., K.

H_{pi} The holding cost per unit per unit time for manufacturer i, where i = 1, 2,..., I.

H_d The holding cost per unit per unit time for the distributor.

H_{rk} The holding cost per unit per unit time for retailer k, where k= 1, 2,..., K.

θ_{rk} The deterioration cost per unit per unit time for retailer k, k= 1, 2,..., K.

θ_d The deterioration cost per unit per unit time for distributor.

θ_{pi} The deterioration cost per unit per unit time for manufacturer i, where i = 1, 2,..., I.

d_k The demand rate for retailer k, where k= 1, 2,..., K.

p_i The production rate for manufacturer i, where i= 1, 2,..., I.

W The stock maximum capacity of distributor.

θ The inventory deterioration rate.

λ_i The proportion of order arrangement for manufacturer i, i.e. $\lambda_{1+ ....+}\lambda_I = 1$.

T The supply cycle time for the producers.

t_d The processing time for the distributor in the distribution center.

t_r The replenishment cycle time for retailers.

The mathematical model is developed on the basis of the following assumptions:

(1) Only a single-product item is considered.
(2) A single-period inventory is considered.
(3) The demand rate is known and constant.
(4) No shortage is allowed.
(5) There is a constraint, $Q \leq W$, for distributor in the space capacity.
(6) The rate of replenishment in the distributor and each retailer is concurrent and instantaneous.
(7) The delivery from manufacturer to distributor is concurrent and instantaneous..
(8) A constant fraction the on-hand inventory deteriorates and no replacement of deteriorated items is allowed.

4. Mathematical model

We consider the supply chain inventory system consisting of multiple manufacturers, one distributor and multiple retailers as depicted in Figure 1. Their cost structures using integration strategy and independent strategy are developed as follows.

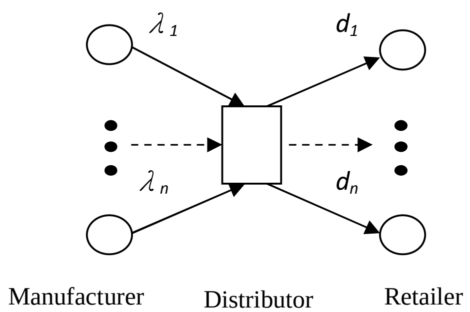

Figure 1. A *n*-manufacturer-*one*-distributor-*n*-retailer network distribution system

4.1 Cost structure of retailer

The change in the retailer's inventory level during an infinitesimal time, *dt*, is a function of the deterioration rateθ, the demand rate d_i and the inventory level $I_{ri}(t)$. The inventory system is formulated as

$$\frac{dI_{rk}(t)}{dt} = -d_k - \theta \cdot I_{rk}(t), \quad 0 \le t \le t_{rk}, \quad k = 1,2,...,K. \quad (1)$$

The various boundary conditions are

$$I_{rk}(0) = q_{rk}, \quad I_{rk}(t_{rk}) = 0, \quad k = 1,2,...,K. \quad (2)$$

After adjusting for the constant of integration, the solution of the above differential equations is

$$I_{rk}(t) = \frac{d_k}{\theta}\left[e^{\theta(t_r - t)} - 1\right] \quad ,0 \le t \le t_r \quad (3)$$

From (15) and (16), the initial inventory level at retailer *k* can be derived as

$$q_{rk} = \frac{-d_k + d_k \exp(\theta t_r)}{\theta} \quad (4)$$

By summing the ordering cost, the holding cost and the deterioration cost, the retailer's total cost per unit time can be obtained as:

$$TC_r(t_r) = \sum_{k=1}^{K}\left[\begin{array}{l} \dfrac{C_{rk}}{t_r} + \dfrac{d_k e^{\theta t_r} - d_k - \theta d_k t_r}{t_r \theta^2} \times H_{rk} \\ + \dfrac{-d_k + d_k e^{\theta t_r} - \theta d_k t_r}{t_r \theta} \times \vartheta_{rk} \end{array}\right] \quad (5)$$

4.2 Cost structure of distributor

The distributor applies a risk pooling in the network inventory model. The change in the distributor's inventory level during an infinitesimal time, *dt*, is a function of the deterioration rate θ and the inventory level $I_d(t)$ as shown in Figure 1 is

$$\frac{dI_d(t)}{dt} = -\theta I_d(t), \quad 0 \le t \le t_d \quad (6)$$

The various boundary conditions are

$$I_d(0) = Q \quad (7)$$

After adjusting for the constant of integration, the solution of the above differential equations is

$$I_d(t_d) = \sum_{k=1}^{K} q_{rk} \cdot \exp(-\theta \cdot t_d), \quad 0 \le t \le t_d \quad (8)$$

From (7) and (8), the initial inventory level at distributor can be obtained as

$$Q = \sum_{k=1}^{K} q_{rk} \cdot \exp\left(\theta \cdot t_d\right) \quad (9)$$

where t_d is assumed as the process time in the distribution center.

By summing the ordering cost, the transportation cost, the holding cost and the deterioration cost, the distributor's total cost per unit time can be obtained as:

$$TC_d(Q) = \frac{1}{t_d} \cdot \left(\sum_{k=1}^{K} C_{dk} + \sum_{k=1}^{K} C_{fk} + \frac{H_d}{\theta}\left[Q - Q\exp(-\theta \cdot t_d)\right] + \vartheta_d\left(Q - Q\exp(-\theta \cdot t_d)\right) \right) \quad (10)$$

4.3 Cost structure of manufacturers

The change in the manufacturer's inventory level during an infinitesimal time, *dt*, is a function of the deterioration rate θ, production rate p_i and the inventory level $I_{pi}(t)$. The inventory system is formulated as:

$$\frac{dI_{pi}(t)}{dt} = p_i - \theta I_{pi}(t), \quad 0 \le t \le t_p, \quad i = 1,2...,n. \quad (11)$$

The various boundary conditions are

$$I_{pi}(0) = 0, \quad I_{pi}(t_p) = q_{pi} = \lambda_i Q \tag{12}$$

After adjusting for the constant of integration, the solution of the above differential equations is

$$I_{pi}(t) = \frac{p_i\left(1 - \exp(-\theta t)\right)}{\theta}, \quad 0 \le t \le t_p \tag{13}$$

From (12) and (13), the initial inventory level at manufacturer *i* can be obtained as

$$q_{pi} = \lambda_i Q = \frac{p_i\left(1 - \exp(-\theta \cdot t_p)\right)}{\theta} \tag{14}$$

Let A_{pi} as the carrying inventory of the manufacturer i. By summing the transportation cost, the holding cost and the deterioration cost, the manufacturers' total cost per unit time can be obtained as:

$$TC_p = \frac{1}{t_p}\sum_{i=1}^{I}(C_{pi} + A_{pi} \cdot H_{pi} + A_{pi}\theta \cdot P_{pi}) \tag{15}$$

One has

$$TC_p(t_p) =$$

$$\sum_{i=1}^{I}\frac{1}{t_p}\left\{\begin{array}{l} C_{pi} + \dfrac{1}{\theta}\left[p_i\left(\dfrac{1}{\theta}\ln\left[\dfrac{p_i}{p_i - q_{pi}\theta}\right]\right) - \dfrac{p_i\left(1 - \exp(-\theta \cdot t_p)\right)}{\theta}\right]H_p \\ + \left(p_i\left(\dfrac{1}{\theta}\ln\left[\dfrac{p_i}{p_i - q_{pi}\theta}\right]\right) - \dfrac{p_i\left(1 - \exp(-\theta \cdot t_p)\right)}{\theta}\right) \times \vartheta_p \end{array}\right\} \tag{16}$$

4.4 Joint cost structure

There are players in the network inventory of the three-echelon supply chain. The optimization problem is a constrained nonlinear program, stated as:

$$\textit{Minimize} \qquad TC = TC_r + TC_d + TC_p \tag{17}$$

$$\textit{Subject to}: \quad Q \le W, \; n \in N$$

$$0 \le t_r$$

The inventory cost function *TC*is a function of the independent variables t_r. Due to the complexity of the expressions and the un-interrelated parameters, it is not possible to prove the convexity of $TC(t_r)$ analytically. The classical iterative optimization techniques very often results in local optimum present. In this study, the optimum value of t_r for minimum $TC(t_r)$ is is obtained by using SA as described below.

5. The proposed simulated-annealing solving approach

5.1 Simulated annealing

Consider an ensemble of molecules at a high temperature, which are moving around freely. Since physical systems tend towards lower energy states, the molecules are likely to move to positions that lower the energy of the ensemble as a whole as the system cools. However molecules actually move to positions that increase the energy of the system with a probability $e^{-\Delta E/T}$, where ΔE is the increase in the energy of the system and T the current temperature. If the ensemble is allowed to cool slowly, it will eventually promote a regular crystal, which is the optimal state rather than flawed solid, which would be the poor local minima. In function optimization, a similar process can be defined. This process can be formulated as the problem of finding, among a potentially very large number of solutions, a solution with minimum cost. By establishing the correspondence between the cost function and the free energy and between the solutions and the physical states, a solution method was introduced (Kirkpatrick, Gelatt and Vecchi [8, 9], Cerny [3]) in the field of optimization based on a simulation of the physical annealing process. This method is called Simulated Annealing. The Simulated Annealing algorithm to solve such problems is given below [15]:

1. Start with some state S.
2. $T=T_0$
3. Repeat {
4. While (not at equilibrium){
5. Perturb S to get a new state Sn
6. $\triangle E=E(S_n)-E(S)$
7. If・$\triangle E<0$
8. Replace S with S_n
9. Else with probability $e^{-\Delta E/T}$
10. Replace S with S_n
11. }
12. T=C*T /* 0<C< 1 */
13. } Until (frozen)

In this algorithm, the state (S) becomes the state (approximate solution) of the problem in question rather than the ensemble of molecules, energy (E) corresponds to the quality of S and is determined by a cost function used to assign a value to the state, temperature (T) is a control parameter used to guide the process of finding a low cost state, T_0 is the initial value of T and C (0<C<1) is a constant used to decrease the value of T. To apply SA to a specific problem, we must define a cooling or annealing schedule for the algorithm, a perturbation function and an energy function. Any annealing schedule should include the initial temperature, the rate at which the temperature should be decreased and good termination conditions for both the loops of the algorithms.

5.2 SA procedures

In the scenario under investigation, the problem is to minimize the joint cost $TC(t_r)$. The optimum values of t_r and n for which $TC(t_r)$ is minimum are obtained using SA process as described below.

Step 1. Representation and initialization: a real variable t_r is used to represent the optimum replenishment cycle time for retailers. A real constant t_{r0} satisfying problem constraint is randomly generated and taken as the initial guess of t_r.

Step 2. Perturbation function: a random number r between -0.25 and +0.25 is generated using random number generator. t_r+r is taken as neighbour solution of t_r if t_r+r satisfies the constraints of the problem.

Step 3. Energy function: our problem is to find the optimum inventory level t_r such that joint cost $TC(t_r)$ is minimum. Here $-TC(t_r)$ is taken as the energy function of the solution t_r.

Step 4. Cooling schedule: initial temperature T_0 is taken according to different parameter values of the energy function and reducing factor C for T (temperature) is taken as 0.99.

Step 5. Repeat steps 1 ~ 4 for all possible n values until the minimum $TC(t_r^*)$ is found.

6. Numerical example

A numerical example is used to illustrate the theory developed in the study. The related data of the manufacturers are as follows:

W=6,000 units/year,

p_i={9,000 10,000 11,000 12,000 13,000} units/year,

H_{pi1}={5, 6, 7, 8, 9 }$/year,

C_{pi}={4,000 5,000 6,000 7,000 8,000} $/delivery,

P_{pi}={20, 25, 30, 35, 40} per $/unit,

λ_i=[0.2, 0.7].

The related data of the distributor are as follows: H_d= $20/unit/year,

C_{di}={200, 250, 300, 350, 400} $/ time,

P_d=$30 per unit,

C_{fi}={4,000 4,500 5,000 5,500 6,000}$/pdelivery,

The related data of the retailers are as follows:

d_k={6,000 7,000 8,000 9,000 10,000}units/year,

H_{rk}={50, 60, 70, 80, 90} $/per unit per year,

C_{rk}={200, 300, 400, 500, 600}$/time,

P_{rk} ={50, 60, 70, 80, 90} per $/unit,

θ=0.05.

Using the integrated approach, the minimal total cost is computed. The optimal solutions can be obtained by SA method. The optimal solutions are as shown in Table 1.

Table 1. Analysis of the result under various combination

Description	Integrated modeling				
	(i,j,k)=(1,1,1)	(i,j,k)=(2,1,2)	(i,j,k)=(3,1,3)	(i,j,k)=(3,1,4)	(i,j,k)=(4,1,5)
t_r^*	0.12	0.14	0.20	0.41	NA
t_{p1}^*	0.075	0.075	0.085	0.093	NA
t_{p2}^*	NA	0.099	0.095	0.104	NA
t_{p3}^*	NA	NA	0.121	0.121	NA
t_{p4}^*	NA	NA	NA	0.146	NA
q_{p1}^*	1230	906	1016	1243	NA
q_{p2}^*	NA	1359	1227	1304	NA
q_{p3}^*	NA	NA	1408	1408	NA
q_{p4}^*	NA	NA	NA	1672	NA
q_r^*	1214	2168	3540	5740	NA
Q^*	1226	2204	3604	5888	NA
TC	34843	348437	499425	1009425	NA
$PICR$(%)	96.55	65.48	50.52	NA	NA

Notes: *PICR*: Percentage of Integration Cost Reduction

$$PICR=\left\{ \underset{(3,1,4)}{TC}(t_r^*)-\underset{(i,j,k)}{TC}(t_r^*)\right\} / \underset{(3,1,4)}{TC}(t_r^*)$$

7. Sensitivity analysis

With the integrated policy, the optimal values of t_r, TC_r, TC_d, TC_p, TC and for a fixed set of parameters Φ = {d_1, d_2, H_{r1}, H_{r2} C_{r1}, C_{r2}, P_{r1}, P_{r2}, H_d, C_{d1}, C_{d2}, P_d, C_{f1}, C_{f2}, p_1, p_2, H_{p1}, H_{p2}, C_{p2}, C_{p2}, P_{p1}, P_{p2},θ}are denoted by n^*, t_r^*, TC_r^*, TC_d^*, TC_p^* and TC^* respectively. The changes in n^*, t_r^*, TC_r^*, TC_d^*, TC_p^* and TC^* are then considered when the parameters in the set Φ vary. Sensitivity analysis where the parameters in the set Φ changes by { -30%, +30%}are carried out.

The main conclusions from the sensitivity analysis are as follows:

(1) When the retailer's demand rate increases, each player's total cost, except the retailer, tends to increase. The joint total cost increases as well.

(2) As the distributor's holding cost increases, the number of deliveries from the distributor to the retailer tends to increase. The distributor's total cost and the joint total cost TC tend to increase.

(3) As the retailer's holding cost increases, each player's total cost, except the retailer, tends to increase. The joint total cost increases as well.

(4) The joint total cost TC is more sensitive to the parameter of the distributor's holding cost H_d. When H_d decreases and increases by 30%, TC tends to change from -11% to

9%.

(5) When the deterioration rateθincreases, the number of deliveries from the distributor to the retailer tends to increase. The distributor's total cost and the joint total cost *TC* tend to increase.

8. Conclusions

This study discusses the optimal joint-cost policy in a two-manufacturer two-retailer and three-echelon supply chain inventory model that integrates the upper, middle, and lower levels of the supply chain. By using the integrated approach that takes account of the manufacturers, the distributor, and the retailers, the total joint cost is found to be less than an independent approach by the individual players. The result is validated through sensitivity analysis. Again for the first time, the multi-echelon supply chain inventory problem has been solved by SA algorithm which always ensures global optimum. Furthermore, recent advances in communications and information technology provide greater opportunity for significant savings in the costs of logistics by implementing strategic alliances within the marketing channel. The model has potential application in a multi-echelon supply chain inventory system.

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