Optimal Deteriorating Inventory Models for Varies Supply Life Cycles

Gede Agus Widyadana[1*], Kun Jen Wang[**], Nyoman Sutapa[*]

[*]Industrial Engineering Department, Petra Christian University, Surabaya, Indonesia
[**]Industrial Management, National Taiwan University of Science and Technology, Taipei, Taiwan
[1]gedeaw@gmail.com

***Abstract*— Agriculture items, such as fruits and vegetables, have different supply and demand characteristics during a harvest period. Fruits supply in the first and end of harvest time are not reliable so sometimes supply are not available when needed. Fruits demand is different during harvest season. In the first harvest season, demand depends on price and at the end of harvest time, the demand depends on presentation of the items. In this study, inventory deteriorating items models for the first and the end of the harvest season are developed. Since closed-form solutions cannot be derived from the models, a Genetic Algorithm and a heuristic method are used to solve the problems. A numerical example and sensitivity analysis are conducted to illustrate the model and get insights. The sensitivity analysis shows that the supplier will increase his price when supply is not reliable at the early harvest period. The results show that the unreliable supply is susceptible to the total cost at the end of the harvest period.**

Keywords— *Inventory, Deteriorating Items, Unreliable Supply, Price Dependent, Optimization*

1. INTRODUCTION

Efficiency has full attention in most companies since efficiency is one competitive advantage for any companies to survive and compete. Agriculture companies has higher effort to increase their efficiencies since their items are decay or spoilage gradually within time. Their products such as fruits and vegetables can be included as deteriorated items. Many fruits and vegetables have specific harvest period and different life supply. In the first harvest time not so many items can be ripped, so only a few stocks are available in the market and the price is high. In this period, customers are sensitive with the price. They consideration is to wait until stock available enough and price reduced or buy at current price. In the second period, most trees are ready to be harvested, so the stock is enough and demand is stable. In this stable condition the model using a general deteriorating inventory model. In the last harvest season, only a few trees still productive and can be harvested. In this period, the stock less and only a few good items available in the market. The supplier has problems to collect the items, so sometimes they delay their shipment. Usually, the customer wants to buy when the items which available on the shelf are interesting and has enough quantity. In this phase, a stock dependent demand with unavailability supply model can be used.

Stock dependent inventory model was developed first by Gupta and Vrat [1] then Mandal and Phaudjar [2] developed an economic production quantity model for deteriorating items by considering stock-dependent consumption rate. Baker and Urban [3] developed inventory deteriorating items models with stock dependent demand. Pal et. al [4] continued the work of Baker and Urban [3] . Hou [5] developed deteriorating inventory model with stock dependent demand and consider inflation and time discounting since in countries that suffered from large-scale inflation and purchasing power declines, the effects of inflation and time value of money should be considered. Lee and Dye [6] developed a deteriorating inventory items model with stock dependent demand and considering investment cost for controlling deteriorating rate. They assumed that deteriorating rate could-be controlled using various effort by investing in technology. Giri et al. [7] extended the excellent work of Urban [8] by setting the constant deteriorating rate. Dynamic pricing and periodic order quantity model for deteriorating items with stock-dependent demand were developed by Li et al. [9]. In their model, they allowed shortages, and the backlogging rate is variable and dependent on the duration for the next replenishment. Research on deteriorating stock dependent model are continued by Teng and Chang [10]. They introduced a production economic quantity model for deteriorating items with great goods displayed in a supermarket could make more sales and profit. The demand depends on stock and the selling price. They assumed that the number of stock on display is limited by the number of shelf or display space. They mentioned that a considerable amount of stock on display is not good because too much stock gives a negative impression to customers. Pricing is an important issue in deteriorating inventory model since a higher price can reduce demand and increase

International Journal of Supply Chain Management
IJSCM, ISSN: 2050-7399 (Online), 2051-3771 (Print)

deteriorate items. This is the research focus of Das et al. [11]. They assumed that demand is depleted due to deterioration and demand. They considered price discounts and delayed payments to increase demand. Teng et al. [12] developed deteriorating inventory models with time-varying deterioration rate, allowed shortage and percentage of the shortage items are backlogged. They considered an advance payment since the buyer needs to pay the acquisition cost in advance as a deposit for seasonal items. Deteriorating inventory model by considering pricing and discount to increase sales and profit was developed by Banerjee and Agrawal [13]. The demand was considered depend on price and freshness. Musa and Sani [14] developed an inventory model with delayed deterioration. The items do not deteriorate at once when they are stocked. They assumed that supplier gives delay in payment for the buyer to boost sales. A model for the supplier-retailer-customer supply chain with partial trade credits to their customers was developed by Tiwari et al. [15]. All players should figure the optimal selling price to optimise their profits. Some research have been a focus on stock dependent demand and it is relevant for fruits and vegetables. Consumers tends to attract to buy when they find a pile of fresh fruits or vegetables. Chakraborty et al. [16] developed supply chain deteriorating items model by considering stock dependent demand and multi items. They assumed that the supply chain environments are fuzzy. Fruits and vegetables demands mostly varies in time therefore Balkhi and Bankherouf [17] developed a deteriorating inventory model by considering stock dependent and time-varying demand. Zhang et al. [18] developed non-instantaneous deteriorating inventory items where customer demand depends on the number of the items displayed in the store and the sales price. Pando et al.[19] developed an optimal profit of inventory deteriorating items with stock dependent demand and nonlinear holding cost. The holding cost depends on time and stock level. Tiwari et al. [20] developed a deteriorating inventory model for a two-echelon supply chain by considering the limitation of the display area. They assumed demand rate is dependent on delayed stock price and displayed stock level. Sometimes demand not only depend on one factor but depends on three factors which are stock level, time and price (Liuxin et al. [21]). Duan et al. [22] developed dynamic pricing for economic production deteriorating inventory items model with price dependent demand and stochastic process demand. Later, Hsieh and Dye [23] developed an inventory lot size model for deteriorating items by considering price and time demand function. They considering partial -backlogging and the price is periodically adjusted upward and downward.

Although there is intensive research on deteriorating inventory model and optimisation, only a few considering the supply life cycle of items such as fruits and vegetables. Many seasoned fruits and vegetables have supply life cycle. At the first harvest time, only a few fruits can be harvested, so the price tends to be high, and customer demand depends on the price offered by the supplier the supply is not always available. In the second phase which is the harvest phase, the price and supply tend to be stable since more fruits are available. When the supply decrease, the amount of harvested fruits are less and some items on a shelf is less than the earlier period, and the fruits are not as interesting as harvest period, so customer's demand depends on a number of stocks at the store shelf. This paper contributes by developing deteriorating inventory models for different supply life cycles, so retailers can adapt their strategies depend on the items life cycle to optimise their profit. This paper is divided into four sections. In the first section, relevant literatures are introduced, and the contribution of the paper is shown. Mathematical models are developed in Section 2, and a numerical example and sensitivity analysis are presented in Section 3 to give management insight into the model. In the last section, exciting conclusions are shown

2. MODEL DEVELOPMENT

In this study we develop two deteriorating inventory models for describing two condition in first harvest stage and the last period of harvest stage. The preliminary harvest stage can be seen in Figure 1. The replenishment period depend on the product price since demand depend on price. Most price dependent demand model assume replenished item available directly when needed, but many conditions such as first harvest time, stock is not stable. There is a possibility that items have not arrived when needed and result in a shortage in T_d period. The inventory level in the last harvest period in shown in Figure 2. Demand in the last harvest period depend on inventory level in a period since at this period not many items have good conditions and consumer tend to buy when they see fresh items. When the stock is high, demand is high and stock deplete quickly. Supply in the last harvest time is similar as preliminary harvest period when supply is not constant, so items have not arrived when needed and result in shortage for T_d period.

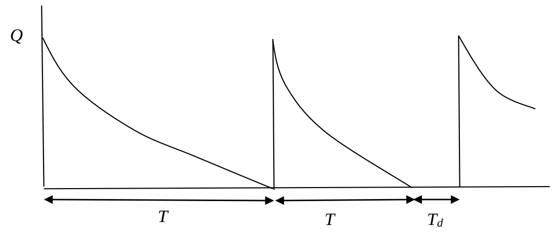

Figure 1. Inventory level for preliminary harvest period

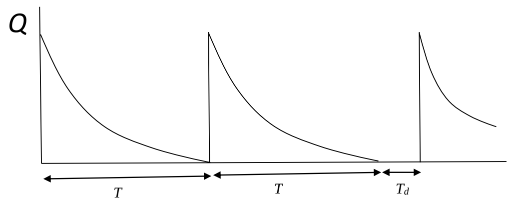

Figure 2. Inventory level in the last harvest period

Notations:

I_t	= Inventory level at t period
p	= price rate
α	= price constant rate
ε	= price sensitivity rate
β	= stock sensitivity rate
d	= demand rate
θ	= deteriorating rate
K	= setup cost
H	= holding cost
S	= lost sales cost
TC	= total inventory cost
T	= total replenishment time
T_d	= shortage period
TP	= total profit

2.1. Deteriorating inventory model with price dependent

In the inventory deteriorating model with price dependent demand, the inventory level depends on the deteriorating rate and demand, where the demand depends on the price rate. The following equation can denote the inventory level in this model:

$$\frac{dI}{dt} + \theta I_t = -\alpha p^{-\varepsilon}, \qquad 0 \le t \le T \quad (1)$$

Since $I_{(0)}=Q$ where t=0, then one has:

$$I_t = \frac{\alpha p^{-\varepsilon}}{\theta}(e^{-\theta(T-t)} - 1) \text{ for } 0 \le t \le T \quad (2)$$

Total inventory for $0 \le t \le T$ is

$$\int_0^T I(t)dt = \int_0^T \frac{\alpha p^{-\varepsilon}}{\theta}(e^{-\theta(T-t)} - 1)\,dt$$

$$= -\frac{\alpha(e^{-\theta T} + \theta T - 1)p^{-\varepsilon}}{\theta^2 p^{\varepsilon}} \quad (3)$$

Total profit can be derived from the total revenue minus cost of placing an order, and cost of carrying inventory. The total profit can be modelled as:

$$TP = p(\alpha p^{-\varepsilon}) - \frac{K}{T} - \frac{HI_t}{T}, \quad (4)$$

substitute I_t from (3) to (4) , one has:

$$TP = p(\alpha p^{-\varepsilon}) - \frac{K}{T} - \frac{H\left(-\frac{\alpha(e^{-\theta T} + \theta T - 1)p^{-\varepsilon}}{\theta^2 p^{\varepsilon}}\right)}{T} \quad (5)$$

2.2. The price dependent demand with lost sales case

The total profit for lost sales consists of total revenue minus setup cost, holding cost and loss of goodwill. Lost sales occur when supply unavailability period is longer than the replenishment period. The total profit can be expressed as:

$$TP = p(\alpha p^{-\varepsilon}) - \frac{K}{E(T)} - \frac{H\left(-\frac{\alpha(e^{-\theta T} + \theta T - 1)p^{-\varepsilon}}{\theta^2 p^{\varepsilon}}\right)}{E(T)} + S\alpha p^{-\varepsilon}\frac{\int_{t=T}^{\infty}(t-T)f(t)dt}{E(T)} \quad (6)$$

Replenishment time consists of replenishment period and supply unavailability time which is longer than replenishment period. The expected replenishment time can be written as:

$$E(T) = T + E(T_d)$$

$$= \left[T + \int_{t=T}^{\infty}(t-T)f(t)dt\right] \quad (7)$$

The total profit can be derived by substituting (7) into (6) as follows:

$$TP(p,T)=\frac{E\left[p(\alpha p^{-\varepsilon})-K-H\left(-\frac{\alpha(e^{-\theta T}+\theta T-1)p^{-\varepsilon}}{\theta^2 p^{\varepsilon}}\right)+S\alpha p^{-\varepsilon}\frac{\int_{t=T}^{\infty}(t-T)f(t)dt}{E(T)}\right]}{T+\int_{t=T}^{\infty}(t-T)f(t)dt} \quad (8)$$

2.3. Uniform distribution case

In this case, we assume that the unavailability time t is a random variable that is uniformly distributed over the interval [0, b]. The uniform probability density function, $f(t)$, is given as:

$$f(t)=\begin{cases}1/b, & 0\le t\le b\\ 0, & otherwise\end{cases}$$

For a uniform distribution, the value of T can be expressed as:

$$E(T)=T+\frac{(b-T)^2}{2b} \quad (9)$$

Substitute uniform probability density function in (8), one has:

$$TCT(p,T)=\frac{\left[Kp(\alpha p^{-\varepsilon})-K-H\left(-\frac{\alpha(e^{-\theta T}+\theta T-1)p^{-\varepsilon}}{\theta^2 p^{\varepsilon}}\right)+S\alpha p^{-\varepsilon}\left(\frac{b-T}{b}\right)\right]}{T+\frac{(b-T)}{2T}} \quad (10)$$

Since the closed form solution cannot be found and there are two decision variables. The model is solved using a genetic algorithm and run in Matlab. A simple Genetic Algorithm is used to solve the model and run in Matlab.

Chromosome

Chromosome represents the optimal price (p) and the optimal ordering time (T). The population type is a bit string with 14 cells where seven cells are for the optimal price, and seven cells are for the optimal ordering time.

Initial solution and population

The initial solutions are generated randomly, and the population size is equal to 200.

Selection and reproduction

For each generation, the parent chromosomes are chosen to use roulette wheel where chromosomes with the highest profit have a high probability to be chosen as parents. The reproduction using crossover and mutation. The crossover scheme is two points crossover, and the mutation scheme is uniform with probability 1%. Elitism scheme is using to guarantee the best chromosome for every generation will be a child in the next generation.

Stopping criteria

The genetic algorithm is stopped after 100 generations

2.4. Deteriorating inventory model with stock dependent demand

In the end of harvest time, demand depends on the level of stock shown to the customers. The following equation can denote the inventory level for stock dependent demand deteriorating inventory:

$$\frac{dI}{dt}+\theta I_t=-d(I(t)), \qquad 0\le t\le T \quad (11)$$

One has:

$$I(t)=\frac{1}{\theta+\beta}(e^{(\theta+\beta)(T-t)}-1) \qquad 0\le t\le T$$

Total inventory for $0\le t\le T$ is

$$\int_0^T I(t)dt=\int_0^T\frac{1}{\theta+\beta}(e^{-(\theta+\beta)(T-t)}-1)\,dt$$

$$=\frac{(e^{-(\theta+\beta)T}-\theta T-T\beta-1)}{(\theta+\beta)^2} \quad (12)$$

2.5. The stock dependent demand lost sales case

Since supply is not always available, lost sales could be happened when supply available after replenishment time. So the total inventory cost for lost sales consists of setup cost, holding cost and loss of goodwill. The total inventory cost can be modelled as:

$$TC(T)=E\left[K+h\left(\int_0^T\frac{1}{\theta+\beta}(e^{-(\theta+\beta)(T-t)}-1)\right)+S\frac{\int_{t=T}^{\infty}e^{-\beta t}f(t)dt}{\beta}\right] \quad (13)$$

We can use the same replenishment period as (7), so the total cost per unit time can be derived as follows:

$$TCT(T)=\frac{E\left[K+h\left(\int_{0}^{T}\frac{1}{\theta+\beta}(e^{-(\theta+\beta)(T-t)}-1)\right)+S\frac{\int_{t=T}^{\infty}e^{-\beta t}f(t)dt}{\beta}\right]}{E\left[T+\int_{t=T}^{\infty}(t-T)f(t)dt\right]} \tag{14}$$

2.6. Uniform distribution stock dependent demand lost sales case

Substitute uniform probability density function in (13), one has:

$$TCT(T)=\frac{K+h\left(\frac{(e^{-(\theta+\beta)T}-\theta T-T\beta-1)}{(\theta+\beta)^2}\right)+Sd\left(\frac{(b-T)^2}{2b}\right)}{E(T)} \tag{15}$$

Substitute (9) into (15), one has:

$$TCT(T)=\frac{K+h\left(\frac{(e^{-(\theta+\beta)T}-\theta T-T\beta-1)}{(\theta+\beta)^2}\right)+Sd\left(\frac{(b-T_2)^2}{2b}\right)}{T+\frac{(b-T)^2}{2b}} \tag{16}$$

We derivate (16) in a term of *T* and set the result to be zero to get the optimal *T*.

$$\frac{dTCT(T)}{dT}=\frac{\left(K+h\left(\frac{(e^{-(\theta+\beta)T}-\theta T-T\beta-1)}{(\theta+\beta)^2}\right)+\frac{S\left(\beta^2b^2-2T\beta^2b+2e^{-\beta b}(1+\beta b-T\beta)+\beta^2T^2-2e^{-T\beta}\right)}{2\beta^3b}\right)\left(1-\frac{b-T}{b}\right)}{T+\frac{(b-T)^2}{2b}}$$
$$-\frac{\left(h\left(\frac{(\theta+\beta-(\theta+\beta)e^{T(\theta+\beta)}}{(\theta+\beta)^2}\right)-\frac{S\left(-2\beta^2b+2T\beta^2-2e^{-\beta b}\beta+2\beta e^{-T\beta}\right)}{2\beta^3b}\right)}{T+\frac{(b-T)^2}{2b}}$$
$$=0 \tag{17}$$

Since the closed form solution of (17) cannot be derived, a simple heuristic from Maple is used to solve the model.

3. A NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

A numerical example is shown to illustrate the model. We use $K = 100$, $\theta = 0.05$, $\alpha = 1000$, $\varepsilon = 1.2$, $h = 1$, $S = 10$ and $b = 1$ as a problem for the price dependent model. The Genetic Algorithm is run five times, and the best solution is used. The optimal price (p*) for the problem above is 8, the optimal ordering period (T*) is 0.5078, and the optimal profit is 714.125. The result shows that the optimal ordering period is almost half of the maximum unavailability time. We use similar data for the stock dependent cost problem with an added parameter which is stock dependent parameter $\beta = 10$. Since the closed form solution cannot be found, the model is solved using numerical analysis in Maple. The optimal ordering time (T*) is equal to 0.691 with the optimal cost is 149.23.

A sensitivity analysis is conducted to show the effect of parameters to the decision variables and the fitness value. For the sensitivity analysis, a parameter is changed, and the other parameters are set constant. The sensitivity analysis for price-dependent demand is shown in Figure 2-4. Figure 2 shows the optimal price in varies of parameters. The figure shows price is the most sensitive parameter in varies of price parameters, so is challenging to find the optimal decisions when a product is susceptible to price,. The second parameter which has the biggest effect on price decision is supply unavailability time. The optimal price tends to increase as the supply unavailability time increase. The supplier tries to reduce demand by increasing product price, so it will reduce customer demand and reduce the probability of lost sales products and costs. The optimal price tends to rise as inventory cost increase to increase the supplier's profit. At the other way, the price will be reduced as the ordering cost decrease to get a high demand from the customer.

The price constants parameter is the most sensitive parameter for the optimal replenishment period, so it is challenging to obtain precise optimal replenishment time if the demand is very sensitive to the price. The second most sensitive parameter for the optimal price is the maximum unavailability time. When supply is not reliable, the supplier tries to optimise his profit by increasing their price. This condition is relevant in practice where price tends to high as items are not available enough in the market. Replenishment time has a similar sensitivity pattern with the sensitivity analysis of the price where the most sensitive parameter for replenishment time is the price constants parameter and the second most sensitive parameters is the holding cost and lost sales cost. Replenishment time decrease as inventory cost increase. The supplier tries to optimise his profit by reducing replenishment quantity. The parameter that related to unavailability supply is maximum unavailability time. When the replenishment become more unreliable, shown by a higher maximum unavailability time, the supplier tries to extend the replenishment time result in higher ordering quantity. A similar decision is taken to deal with higher lost sales cost.

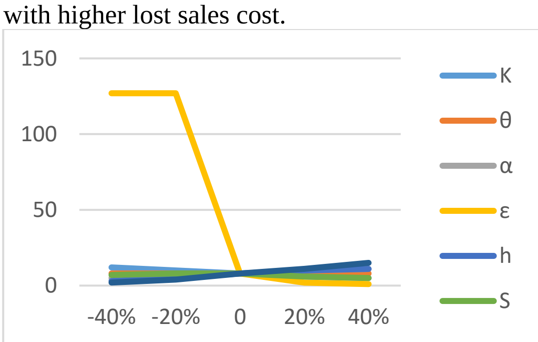

Figure 2. The sensitivity analysis for the optimal price

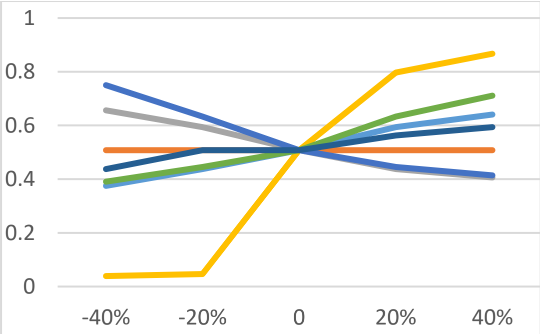

Figure 3. The sensitivity analysis for optimal replenishment time

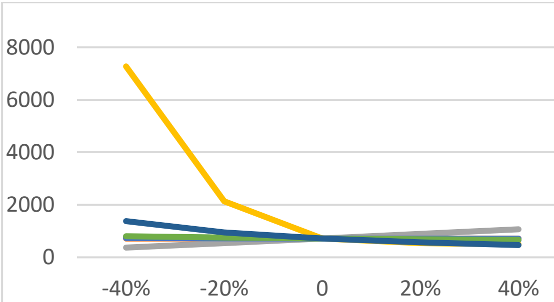

Figure 4. The sensitivity analysis for optimal total profit

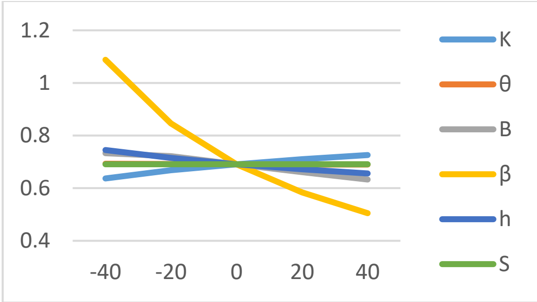

Figure 5. The optimal replenishment time sensitivity analysis for stock dependent demand

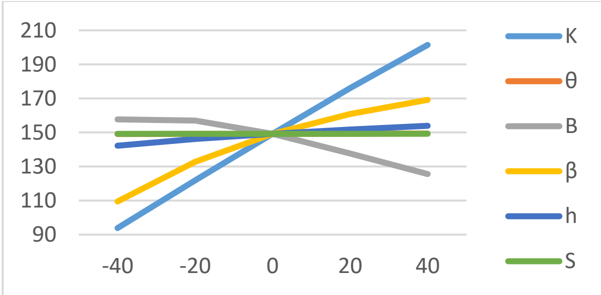

Figure 6. The optimal total profit sensitivity analysis for stock dependent demand

Figure 4 shows the sensitivity analysis for the optimal total profit. It is shown that the most sensitive and the third sensitive parameter to the optimal total profit is the price sensitivity parameter. The profit decrease when the price becomes more sensitive, but the price sensitivity parameter depends on the customer that cannot be managed by the supplier. The second parameter that has a high sensitivity to the total profit is supply unavailability time. The total profit tends to decrease as the supply unavailability time increase. Since the supplier can manage the parameter, the supplier can try to make the supply more reliable to keep the profit high.

The sensitivity analysis for the stock dependent model is shown by analysing the optimal replenishment time and the optimal total cost. Figure 5 shows that the most sensitive parameter to the optimal replenishment time is the stock dependent demand parameter. The optimal replenishment time tends to decrease as the dependent demand parameter increase. When the dependent demand parameter increase, the supplier tries to reduce the total cost by reducing the replenishment time. The optimal replenishment time is susceptible in varies of the unavailability time (b) and ordering cost (K). When the ordering cost increase, the supplier tends to reduce cost by increasing the replenishment time. At the other side, the supplier decreases the replenishment time as the unavailability time increase. The supplier wants to reduce his cost by order fewer items as the reliability increase to cut his lost sales risk.

Figure 6 shows the sensitivity analysis of the optimal total cost in varies of the parameter. It shows that the most sensitive parameters for the optimal total cost are ordering cost (K). The total cost increases significantly as the ordering cost increase, so the supplier should try to reduce their ordering cost to keep their total cost low. The other parameter that significantly affects the optimal total cost is the unavailability time (b) and the stock dependent parameter (β). The total cost tends to decrease as the unavailability time increase. In the other side, the total cost increase as the stock dependent parameters increase. Supply unavailability parameter is sensitive to total cost where the total costs tend to decrease as the supply unavailability time increase.

4. CONCLUSION

Fruits and vegetables have a limited supply period, and they are not available every time in a year. Fruits and vegetables have a period where not many fruits and vegetables ready to be harvested therefore they have a limited supply and high demand. In this phase, the items have price-dependent demand characteristic, where supplier set a high price and customer still sensitive with a new price offered by the supplier. When more fruits and vegetables available in a market, the price becomes stable, and supply can meet customer demand. After the harvest period, supply in the market and customer demand decreasing. Items quality is decreasing, so retailers try to put good products on their shelf. The customer is not too interested in the product since they think they have consumed enough fruits and vegetables. Consumers still interesting to buy the items if they see a pile fruits still available on a shelf with high-quality products. In this phase, customer demand depends on stock they can see in shelf and supply is not always available continually. This period will end when the supplier cannot supply the products again, and we call the whole period as supply life cycle.

Many researchers gave attention to develop deteriorating inventory models, but no paper ever discuss deteriorating inventory items by considering price-dependent demand and unreliable supply for the first period of supply life cycle and considering stock dependent demand and unreliable supply as the last period of supply life cycle. The model is solved using Genetic Algorithm and heuristic methods since closed-form solution cannot be derived. The model is illustrated using a numerical example, and a sensitivity analysis is conducted to show insights 0f the model.

The sensitivity analysis shows that at the first harvesting time, the most sensitive parameters to price decision is price constants parameter and inventory cost. It is more challenging to find the best price where demand is sensitive to price. The second most sensitive parameter to the price is supply unavailability parameters. The supplier will increase his price to reduce cost because of supply unavailability. At the end of the supply period when demand depends on the stock, the total cost mostly sensitive in varies of ordering cost and unavailability supply, so the supplier should keep ordering cost low.

The models show that different period has a different situation and affect the decision to get an optimal profit and cost. At the beginning and the end of harvest period sometimes it is better to deliver items in any quantity in a specific time. Therefore fixed replenishment period model for deteriorating inventory models with price dependent demand and stock dependent demand can be considered being developed for the future research.

REFERENCES

[1] Gupta, R., Vrat, P., ‘Inventory model for stock-dependent consumption rate.’ *Opsearch*, 23, 19-24, (1986)

[2] Mandal B.N. and Phaudjar S., ‘An inventory model for deteriorating items and stock-dependent consumption rate.’ *J. OPl. Res. Soc*., 40, pp, 483-488. (1989)

[3] Baker R.C., and Urban T.L., ‘A deterministic inventory system with an inventory-level-dependent demand rate,’ *J. Opl. Res. Soc.*, 39 no. 9, pp. 823-831. (1988)

[4] Pal S., Goswami A., and Chaudhuri K.S ‘A deterministic inventory model for deteriorating items with stock-dependent demand rate.’ *International Journal of Production Economics*, 32, 291-299. (1993)

[5] Hou K.L ‘An inventory model for deteriorating items with stock-dependent consumption rate and shortages under inflation and time discounting.’ *European Journal of Operational Research*, 168, 463-474. (2006)

[6] Lee Y.P. and Dye C.Y. ‘An inventory model for deteriorating items under stock-dependent demand and controllable deterioration rate.’ *Computers & Industrial Engineering*, 63, 474-482 (2012).

[7] Giri B.C., Pal S., Goswami A., and Chaudhuri K.S ‘An inventory model for deteriorating items with stock dependent demand rate.’ *European Journal of Operational Research*, 95, 604-610. (1996)

[8] Urban, T.L., (1992) ‘An inventory model with an inventory-level-dependent demand rate and relaxed terminal conditions.’ *Journal of the Operational Research Society* 43/7, 721-724

[9] Li Y., Zhang S., and Han J. ‘Dynamic pricing and periodic ordering for a stochastic inventory system with deteriorating items.’ *Automatica*, 76, 200-213. (2017)

[10] Teng J.T., and Chang C.T., (2005) ‘Economic production quantity models for deteriorating items with price- and stock-dependent demand.’ *Computers & Operations Research*, 32, 297-308.

[11] Das D., Roy A., and Kar S., (2010) ‘Improving production policy for deteriorating item under permissible delay in payments with stock-dependent demand rate.’ *Computers and Mathematics with Applications*, 60, 1973-1985

[12] Teng J.T., Cardenas-Barron L.E., Chang H.J., Wu J., and Hu Y. (2016) ‘Inventory lot-size policies for deteriorating items with expiration dates and advance payments.’ *Applied Mathematical Modelling*, 40, 8605-8616.

[13] Banerjee S., and Agrawal S., (2017) ‘Inventory model for deteriorating items with freshness and price dependent demand: Optimal discounting and ordering policies.’ *Applied Mathematical Modelling*, 52, 53-64

[14] Musa A., and Sani B., (2012) ‘Inventory ordering policies of delayed deteriorating items under permissible delay in payments.’ *International Journal of Production Economics*, 136, 75-83.

[15] Tiwari S., Cardenas-Baron L.E., Goh M., and Shaikh A.A., (2018) 'Joint pricing and inventory model for deteriorating items with expiration dates and partial backlogging under two-level partial trade credits in supply chain.' *International Journal of Production Economics*, 200, 16-36.
[16] Chakraborty D., Jana D.K. and Roy T.P., (2015) 'Multi-items integrated supply chain model for deteriorating items with stock dependent demand under fuzzy random and bifuzzy environments.' *Computers & Industrial Engineering*, 88, 166-180.
[17] Balkhi Z.T., and Benkherouf L., (2004) 'On an inventory model for deteriorating items with stock dependent and time-varying demand rates.' *Computers & Operations Research*, 31, 223-240.
[18] Zhang J., Wang Y., Lu L., and Tang W., (2015) 'Optimal dynamic pricing and replenishment cycle for non-instantaneous deterioration items with inventory-level-dependent demand.' *International Journal of Production Economics*, 170, 136-145.
[19] Pando V., San-Jose L., Garcia-Laguna and Sicilia J., (2018) 'Optimal lot-size policy for deteriorating items with stock-dependent demand considering profit maximization.' *Computers & Industrial Engineering*, 81-93.
[20] Tiwari S., Jaggi C.K., Gupta M., Cardenas-Barron L.E., (2018) 'Optimal pricing and lot-sizing policy for supply chain systems with deteriorating items under limited storage capacity.' *International Journal of Production Economics*, 200, 278-290.
[21] Liuxin, C., Xian, C., F.Keblis, M., Gen, L., (2018) 'Optimal Pricing and Replenishment Policy for Deteriorating Inventory under Stock-Level-Dependent, Time-Varying and Price-Dependent Demand.' *Computers & Industrial Engineering* doi: https://doi.org/10.1016/j.cie.2018.06.005
[22] Duan Y., Cao Y., and Huo J., (2018) 'Optimal pricing, production, and inventory for deteriorating items under demand uncertainty: The finite horizon case.' *Applied Mathematical Modelling*, 58, 331-348.
[23] Hsieh T.P., and Dye C.Y., (2010) 'Pricing and lot-sizing policies for deteriorating items with partial backlogging under inflation.' *Expert Systems with Applications*, 31, 7234-7242

.

.

.